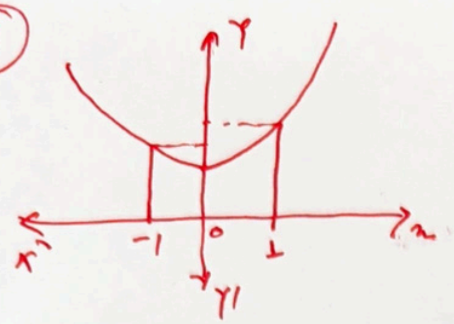
(1)



P(0) is minimum and P(1) is maximum in [-1, 1].

(B)

(2)

$\alpha+\beta = p$

$\alpha\beta = q$

$$\alpha^{1/2}+\beta^{1/2} = \sqrt{\alpha+\beta+2\sqrt{\alpha\beta}}$$
$$= \sqrt{p+2\sqrt{q}}$$

$$\alpha^{1/4}+\beta^{1/4} = \sqrt{(\alpha^{1/4}+\beta^{1/4})^2}$$
$$= \sqrt{\alpha^{1/2}+\beta^{1/2}+2(\alpha\beta)^{1/4}}$$
$$= \sqrt{\sqrt{p+2\sqrt{q}}+2(q)^{1/4}}$$
$$\Rightarrow \sqrt{\sqrt{p+6\sqrt{p}+4q^{1/4}\sqrt{p+\sqrt{4q}}}}$$
$$= \left(p+6\sqrt{p}+4q^{1/4}\sqrt{p+2\sqrt{q}}\right)^{1/4}$$

$\therefore K = 1/4$ (B)

(3) $T_r = \tan^{-1}\left(\frac{1}{2r^2}\right)$

$$= \tan^{-1}\left(\frac{2}{4r^2}\right)$$

$$4r^2 = 1+4r^2-1$$
$$= 1+(2r-1)(2r+1)$$

$$2r+1-(2r-1) = 2$$

$$T_r = \tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r-1)(2r+1)}\right)$$
$$= \tan^{-1}(2r+1) - \tan^{-1}(2r-1)$$

$$\sum_{r=1}^{\infty} T_r = \tan^{-1}(\infty) - \tan^{-1}(1)$$
$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} = a$$

$\tan(a) = \tan\left(\frac{\pi}{4}\right) = 1$ Ans: (A)

4. can express $f(x)$ as

$f(x) = (x-a)^2 \cdot g(x)$ where $g(a) \neq 0$

$$f'(x) = 2(x-a)g(x) + (x-a)^2 g'(x)$$

$$\frac{f'(x)}{f(x)} = \frac{2(x-a)g(x) + (x-a)^2 g'(x)}{(x-a)^2 g(x)}$$

$$= \frac{2}{x-a} + \frac{g'(x)}{g(x)}$$

$$\lim_{x \to a} \left(\lambda \left[\frac{2}{x-a} + \frac{g'(x)}{g(x)} \right] - \frac{1}{x-a} \right)$$

$$= \lim_{x \to a} \left(\frac{2\lambda - 1}{x-a} + \lambda \frac{g'(x)}{g(x)} \right) = m$$

$2\lambda - 1 = 0 \Rightarrow \lambda = \frac{1}{2}$ (C)

5. As both $\vec{P}$ & $\vec{Q}$ have exact same $\hat{k}$ component and it is given that $\vec{P} \parallel \vec{Q}$. so $\vec{P} = \vec{Q}$

$f(t) = -f''(t) \quad g(t) = f'(t)$
$= -g'(t) \quad \Rightarrow f''(t) = g'(t)$

$$|\vec{P}| = \sqrt{(f(t))^2 + (g(t))^2 + 1}$$

$$\frac{d}{dt}[f(t)^2 + g(t)^2]$$

$$= 2f(t)f'(t) + 2g(t)g'(t)$$

(Substituting $f'(t) = g(t)$ and $g'(t) = -f(t)$)

$$= 2f(t).g(t) + 2g(t)(-f(t))$$

$$= 0$$

as $f(t)^2 + g(t)^2$ is a constant, $\sqrt{f(t)^2 + g(t)^2 + 1}$ is also a constant.

(6)

$$\sum_{K=1}^{32}(3K+2)\left\{\sum_{r=1}^{10}\left(\sin\frac{2r\pi}{11}-i\cos\frac{2r\pi}{11}\right)\right\}^K$$

$$\sin\theta - i\cos\theta = -i(\cos\theta + i\sin\theta)$$
$$= -ie^{i\theta}$$

here $\theta = \frac{2r\pi}{11}$

$$\sum_{r=1}^{10} -ie^{i\frac{2r\pi}{11}} = -i\sum_{r=1}^{10} e^{i\frac{2r\pi}{11}}$$
$$= -i(-1) = i$$

[sum of n^{th} roots of unity is zero,

$$1+\sum_{r=1}^{n-1} e^{i\frac{2r\pi}{n}} = 0 \Rightarrow \sum_{r=1}^{n-1} e^{i\frac{2r\pi}{n}} = -1 \text{]}$$

$$\sum_{K=1}^{32}(3K+2)(i)^K$$

from $K=1$ to 4

$5i + 8i^2 + 11i^3 + 14i^4 = 6-6i$

same for $K=5$ to 8

from $K=1$ to 32, 8 such groups

so $S = 8\times 6(1-i)$
$= 48(1-i)$ (D)

(7) Answer : B

⑧

Soln

$t_n \to n^{th}$ term of AP

$t_p = a+(p-1)d = \frac{1}{q}$ —①

$t_q = a+(q-1)d = \frac{1}{p}$ —②

①−②,

$(p-q)d = \frac{1}{q} - \frac{1}{p} \Rightarrow \cancel{(p-q)}d = \frac{\cancel{p-q}}{pq}$

$\Rightarrow d = \frac{1}{pq}$

putting value of d in ①,

$t_p \Rightarrow a + (p-1)\frac{1}{pq} = \frac{1}{q}$

$\Rightarrow a = \frac{1}{q} - \frac{(p-1)}{pq} = \cancel{\frac{1}{q}} - \cancel{\frac{1}{q}} + \frac{1}{pq}$

$\Rightarrow a = \frac{1}{pq}$

$\therefore t_n = \frac{1}{pq} + (n-1)\frac{1}{pq} = \frac{n}{pq}$

now in the eqn $\underbrace{(p+2q-3r)}_{a}x^2+\underbrace{(q+2r-3p)}_{b}x+\underbrace{(r+2p-3q)}_{c}=0$

$a+b+c=0$

$\therefore$ $x=1$ is one root of eqn.

$t_n = 1 \Rightarrow \frac{n}{pq} = 1 \Rightarrow n = pq$

$\therefore$ t_{pq} is root of the eqn

Ans (A)

⑨

Soln.

Total no. of ways to choose 3 no.s from 13 numbers

$= {}^{13}C_3 = \frac{13!}{10! \times 3!} = \frac{11\times 12 \times 13}{3 \times 2} = 11\times 26 = 286$

let the chosen no.s be a, b, c which are in A.P. $(a<b<c)$

$\therefore$ $a + c = \underbrace{2b}_{even}$

$\therefore$ $a+c$ must be even

i.e. either both a & c → even (case 1)

or both a & c → odd (case 2)

Given sequence → $\{1, 2, 3, \ldots, 13\}$

Case ① even $\{2, 4, 6, \ldots, 12\}$

no. of ways to choose both even

${}^6C_2 = \frac{6!}{4!\,2!} = \frac{5\times 6}{2} = 15$

Case ② odd $\{1, 3, 5, \ldots, 13\}$

no. of ways to choose both odd,

${}^7C_2 = \frac{7!}{5!\,2!} = \frac{6\times 7}{2} = 21$

$\therefore$ Total favourable outcomes $= 15 + 21 = 36$

$\therefore$ Probabity of 3 chosen no.s form an AP $= \frac{36}{286}$

$= \frac{18}{143}$ Ans. (B)

(10)

Soln. $f(x) = \frac{1+x}{1-x}$ | Given $A^3 = 0$

$= (1+x)(1-x)^{-1}$ | $\therefore A^3, A^4, \dots = 0$

$f(A) = (I+A)(I-A)^{-1}$

Expansion of $(I-A)^{-1} = I + A + A^2 + \cancel{A^3}^{\,0} + \cancel{A^4}^{\,0} + \dots$

$= I + A + A^2$

$\therefore f(A) = (I+A)(I+A+A^2)$

$= I + A + A^2 + A + A^2 + \cancel{A^3}^{\,0}$

$= I + 2A + 2A^2$

$\therefore f(A) = 1 + 2A + 2A^2$ Ans. (A)

(11)

Soln

(A) if $f(x)$ is decreasing $\Rightarrow \frac{1}{f(x)}$ always $\uparrow$ (given)

$\frac{1}{f(x)}$ is not always increasing

$\because$ while it holds for function which are always +ve /-ve.

it fails when $f(x)$ crosses 0 (where reciprocal is not defined) & changes the sign. $\therefore$ ~~A~~

(B) if $f(x)$ is decreasing $\Rightarrow \frac{1}{f(x)}$ is also decreasing (given)

when $f(x) \downarrow$ then usually $\frac{1}{f(x)}$ is generally increasing (provided sign stays same)

$\therefore$ ~~B~~

(C) if both f and g are +ve functions such that $f \to$ decreasing
$g \to$ increasing

$\frac{f}{g} \to$ decreasing. (given)

This is always true

$\because$ $f(x)$ is decreasing as x increases

& $g(x)$ is $\uparrow$ & +ve (sign stays same)

$\therefore \frac{f(x)}{g(x)}$ numerator is $\downarrow$
& denominator is $\uparrow$

$\therefore$ overall $\frac{f}{g}$ is a decreasing function

Ans (C) ✓

(D) f & g are +ve functions $f \to$ increasing
$g \to$ decreasing

$\frac{f}{g} \to$ decreasing (given)

Similar to case (C) if we approach,

$\dfrac{f\uparrow}{g\downarrow}$ numerator is $\uparrow$ with x

denominator is $\downarrow$ with x

$\therefore \dfrac{f}{g} \rightarrow$ increasing

$\therefore \times$

Ans: (C)

12) $\dfrac{1}{(x-\sin\alpha)} + \dfrac{1}{(x-\sin\beta)} + \dfrac{1}{(x-\sin\gamma)} = 0$

$\Rightarrow (x-\sin\beta)(x-\sin\gamma) + (x-\sin\alpha)(x-\sin\gamma)$
$+ (x-\sin\alpha)(x-\sin\beta) = 0$

$\Rightarrow 3x^2 - x\left(2(\sin\alpha + \sin\beta + \sin\gamma)\right)$

Soln.

$+ (\sin\alpha\sin\beta + \sin\beta\sin\gamma + \sin\alpha\sin\gamma) = 0$

$D = 4(\sin\alpha + \sin\beta + \sin\gamma)^2 - 4\times 3(\sin\alpha\sin\beta + \sin\beta\sin\gamma + \sin\alpha\sin\gamma)$

$= 4(\sin^2\alpha + \sin^2\beta + \sin^2\gamma - \sin\alpha\sin\beta - \sin\beta\sin\gamma - \sin\alpha\sin\gamma)$

$\cancel{4(\sin\alpha(\sin\alpha-\sin\beta) + \sin\beta(\sin\beta-\sin\gamma) + \sin\gamma(\sin\gamma-\sin\alpha))}$

$D = \dfrac{4}{2}(2\sin^2\alpha + 2\sin^2\beta + 2\sin^2\gamma - 2\sin\alpha\sin\beta - 2\sin\beta\sin\gamma - 2\sin\alpha\sin\gamma)$

$= 2\left[(\sin\alpha-\sin\beta)^2 + (\sin\beta-\sin\gamma)^2 + (\sin\gamma-\sin\alpha)^2\right]$

As $D>0$, roots are real & unequal

(A)

(13)

Soln.

(A) $|x-y|<2$

$\because |x-y| = |y-x|$

$|x-y|<2$ is symmetric

But option says neither symmetric

$\therefore$ ~~A~~

(B) $|x| \geq y$

option says reflective, transitive

But it is not transitive

Eg let $x=0$, $y=-5$, $z=2$

$|0| \geq -5$ ✓

$|-5| \geq 2$ ✓

but $|0| \geq 2$ ✗

$\therefore$ ~~B~~

(C) $x>|y|$

option says transitive but neither reflexive / symmetric

reflexive ✗ $\because$ $2>|2|$ is never true

not symmetric $\because$ $2>|5|$ ✗ but $5>|2|$ ✓

Transitive if $x>|y|$ & $y>|z|$

$\because$ since $|y| \geq y$

$\therefore x>|y|>y>|z|$

i.e $x>|z|$

$\therefore$ opt (C) ✓

(D) $x-y<2$

option says that it is symmetric

but eg. $0-2<2$ ✓ but $2-0<2$ ✗ $\therefore$ ~~D~~

Ans: (C)

(14)

Soln. $\displaystyle\int \frac{\operatorname{cosec}^2 x - 2010}{\cos^{2010} x}\,dx = F(x)$

$= \displaystyle\int \frac{\operatorname{cosec}^2 x}{\cos^{2010} x}\,dx - \int 2010 \sec^{2010} dx$

using by parts in I

1st function $\frac{1}{\cos^{2010} x}$ i.e $\sec^{2010} x$

& 2nd function $\operatorname{cosec}^2 x$

$\therefore \sec^{2010} x \int \operatorname{cosec}^2 x\,dx - \int 2010 \sec^{2009} \sec x \tan x \int \operatorname{cosec}^2 x$

$\Rightarrow \sec^{2010} x(-\cot x) - 2010 \int \sec^{2010} \cancel{\tan x} (-\cancel{\cot x})\,dx$

$\Rightarrow \sec^{2010} x(-\cot x) + 2010 \int \sec^{2010} dx = I$

$f(x) = I - \int 2010 \sec^{2010} dx$

$= \sec^{2010} x(-\cot x) + \cancel{\int 2010 \sec^{2010} dx} - \cancel{\int 2010 \sec^{2010} dx}$

$= \dfrac{-\cot x}{\cos^{2010} x} + C$

$\therefore f(x) = \cot x \quad g(x) = \cos x$

$f(\frac{\pi}{4}) = 1 = \cot(\frac{\pi}{4})$

now, $\dfrac{f(x)}{g(x)} = \{x\}$

$\Rightarrow \dfrac{\cot x}{\cos x} = \{x\} \Rightarrow \operatorname{cosec} x = \{x\}$

Range of $\operatorname{cosec} x \Rightarrow (-\infty, -1] \cup [1, \infty)$

Range of $\{x\} \Rightarrow [0, 1)$

$\therefore$ no soln. for $\frac{f(x)}{g(x)} = \{x\}$

Ans (C) 0

15)

$y^2 = 4x \quad \therefore 4a = 4$
$a = 1$

The eqⁿ of a normal to the parabola at any pt. $P(t^2, 2t)$ is

$y + tx = 2at + at^3$

$\Rightarrow y + tx = 2t + t^3$ —— (1)

The eqⁿ of a chord of the parabola $y^2 = 4x$ having mid-point $(h, k) \Rightarrow T = S_1$

$yk - 2(x+h) = k^2 - 4h$

$\Rightarrow yk - 2x = k^2 - 2h \Rightarrow 2x - ky = 2h - k^2$ —— (2)

$\because$ both eqn (1) & (2) represent same line their coefficients must be proportional

$\frac{t}{2} = \frac{1}{-k} = \frac{2t + t^3}{2h - k^2}$

$\therefore t = \frac{-2}{k}$ | $\frac{1}{-k} = \frac{2t + t^3}{2h - k^2}$

putting $t = -\frac{2}{k}$

$\Rightarrow -\frac{1}{k} = \frac{-\frac{4}{k} - \frac{8}{k^3}}{2h - k^2}$

$\Rightarrow \frac{1}{k} = \frac{4k^2 + 8}{k^3(2h - k^2)}$

$\Rightarrow k^2(2h - k^2) = 4k^2 + 8$

$\Rightarrow 2h - k^2 = 4 + \frac{8}{k^2}$

$\Rightarrow h = \frac{k^2}{2} + 2 + \frac{4}{k^2}$

$\Rightarrow h - 2 = \frac{k^2}{2} + \frac{4}{k^2}$

$\Rightarrow x - 2 = \frac{y^2}{2} + \frac{4}{y^2}$

in quesn. given $x - \lambda = \frac{\mu}{y^2} + \frac{y^2}{\nu}$

$\therefore \lambda = 2 \quad \mu = 4 \quad \nu = 2$

$\therefore \lambda + \mu + \nu = 2 + 4 + 2$

$= 8$ Ans. (A)

16) $\Rightarrow 0 \leq \sin^2 x \leq 1$

$\Rightarrow 1 \leq 1 + \sin^2 x \leq 2$

$\Rightarrow \frac{1}{2} \leq \frac{1}{1+\sin^2 x} \leq 1$

$\Rightarrow \sin^{-1}\left(\frac{1}{2}\right) \leq \sin^{-1}\left(\frac{1}{1+\sin^2 x}\right) \leq \sin^{-1}(1)$

$\Rightarrow \frac{\pi}{6} \leq \sin^{-1}\left(\frac{1}{1+\sin^2 x}\right) \leq \frac{3\pi}{6}$

$K \in [1, 3]$ (D)

17)

(17)

Total no. of mappings from A to A $= n^n$

Number of injective mappings $= n!$

Probability $= \frac{n!}{n^n}$

$\frac{n!}{n^n} = \frac{3}{32} \Rightarrow n = 4$ (D)

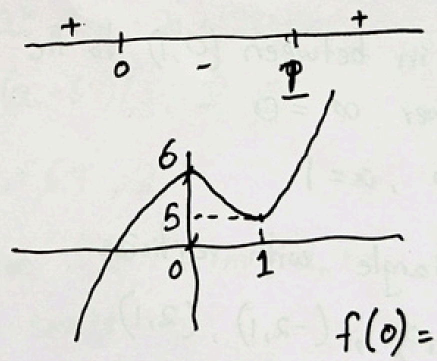
18) $f(x) = 2x^3 - 3x^2 + 6$

For inverse to exist $\rightarrow$ function must be bijective in the interval

$\Rightarrow$ A function, if continuous, to be bijective, it should be strictly increasing or decreasing

$f'(x) = 6x^2 - 6x$

$= 6x(x-1)$



$f(0) = 6$, $f(1) = 5$

The Domain is given as $[a, \infty)$,

Smallest value of $a = 1$

Co-domain $\rightarrow [5, \infty)$ (C)

19)

$$a = \lim_{n\to\infty} (\cos^2 x)^n$$

for $x = n\pi \rightarrow \cos x = \pm 1$

$$a = \lim_{n\to\infty} (1)^n = 1$$

$$b = \lim_{n\to\infty} (\cos^2 x)^n$$

for $x \neq n\pi \rightarrow$ $\cos x$ is less than $\underline{1}$

Anything in between $(0,1)$ to the power $\infty = 0$

$\therefore b = 0 \ , a = 1$

Area of triangle with verticies $(1,0), (-2,1), (2,1)$

Using determinant

$$\frac{1}{2} \times \begin{vmatrix} 1 & 0 & 1 \\ -2 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \frac{1}{2} \times |(-4)| = 2 \text{ sq unit}$$

A

20)

Since OACB is a rectangle with adjacent sides along $\bar{a}$ & $\bar{b}$, $\bar{a}$ and $\bar{b}$ are perpendicular

$$|\bar{a} \times \bar{b}| = |a||b| \sin 90^\circ = ab$$

$$16 |\bar{a} \times \bar{b}| = 3(|\bar{a}| + |\bar{b}|)^2$$

$$\Rightarrow 16ab = 3a^2 + 3b^2 + \cancel{6ab} \ 6ab$$

$$\Rightarrow 3a^2 + 3b^2 - 10ab = 0$$

$$(3a - b)(3b - a) = 0$$

This gives two cases $\rightarrow b = 3a$, $\rightarrow a = 3b$

$$\bar{OC} = \bar{a} + \bar{b}$$

$$\bar{AB} = \bar{b} - \bar{a}$$

$$\cos\theta = \left| \frac{\bar{OC} \cdot \bar{AB}}{|\bar{OC}| |\bar{AB}|} \right|$$

$$\bar{OC} \cdot \bar{AB} = (\bar{a} + \bar{b}) \cdot (\bar{b} - \bar{a}) = b^2 - a^2$$

$$|\bar{OC}| = |\bar{AB}| = \sqrt{a^2 + b^2}$$

$$\cos\theta = \left| \frac{b^2 - a^2}{a^2 + b^2} \right|$$

Substitute $b = 3a$

$$\cos\theta = \frac{8a^2}{10a^2} = \frac{4}{5} \qquad \tan\frac{\theta}{2} = \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = \frac{1}{3}$$

A

21)

$\Rightarrow \bar{r} \times \bar{a} = \bar{b} \times \bar{a}$

$\Rightarrow \bar{r} \times \bar{a} - \bar{b} \times \bar{a} = 0$

$\Rightarrow (\bar{r} - \bar{b}) \times \bar{a} = 0$

As cross product is zero, the vectors $\bar{r} - \bar{b}$ is parallel to $\bar{a}$

$$\bar{r} - \bar{b} = \lambda \bar{a}$$

$\bar{r} = \bar{b} + \lambda \bar{a}$ ----(1)

$\Rightarrow \bar{r} \times \bar{b} = \bar{a} \times \bar{b}$

$\Rightarrow (\bar{r} - \bar{a}) \times \bar{b} = 0$

$\bar{r} = \bar{a} + \mu \bar{b}$ ---(2)

Equating equations (1) and (2)

$\bar{b} + \lambda \bar{a} = \bar{a} + \mu \bar{b}$

$(\lambda - 1)\bar{a} = (\mu - 1)\bar{b}$ ----(3)

As $\bar{a} = \hat{i} + \hat{j}$, $\bar{b} = 2\hat{i} - \hat{k}$ are not parallel, equation 3 is satisfied only when LHS = RHS = 0, so

$\lambda - 1 = 0$, $\lambda = 1$

$\mu - 1 = 0$, $\mu = 1$

$\bar{r} = \bar{a} + \bar{b} = 3\hat{i} + \hat{j} - \hat{k}$ (D)

22)

Let GP be, $a, ar, ar^2 \ldots\ldots ar^{n-1}$

$a_1 + a_1 r^{n-1} = 66$ ----(1)

$a_1 r \times a_1 r^{n-2} = 128$ ----(2)

$\Rightarrow a_1^2 r^{n-1} = 128$

$\Rightarrow a_1(a_1 r^{n-1}) = 128$

$\Rightarrow a_1(66 - a_1) = 128$

$\Rightarrow a_1^2 - 66a_1 + 128 = 0$

$\Rightarrow a_1^2 - 64a_1 - 2a_1 + 128 = 0$

$\Rightarrow (a_1 - 64)(a_1 - 2) = 0$

$a_1 = 64, 2$

Since for $n > m$, $a_n > m$. So a_1 must be smaller

$a_1 = 2$ $\quad a_1 r^{n-1} = 64$

$r^{n-1} = 32$

$\sum_{r=1}^{n} a_r = 126$

$\Rightarrow \dfrac{a(r^n - 1)}{r - 1} = 126$

$\Rightarrow \dfrac{r \cdot r^{n-1} - 1}{r - 1} = 63$

$\Rightarrow 32r - 1 = 63r - 1$

$\Rightarrow r = 2 \qquad 2^{n-1} = 2^5$, $n = 6$

(C)

23) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Equation of Tangent to the ellipse

$y = mx \pm \sqrt{a^2m^2+b^2}$

$y = mx \pm \sqrt{4m^2+9}$ ---→ Tangent

<u>Perpendicular Distance from Center</u>

31) $x^2+y^2 = 5^2$

Center → $(0,0)$

Radius → 5

$$d = \left|\frac{0 - 0 \pm \sqrt{4m^2+9}}{\sqrt{m^2+(-1)^2}}\right| = \sqrt{\frac{4m^2+9}{m^2+1}}$$

<u>Intercept made by circle</u>

$L = 2\sqrt{R^2-d^2}$

To minimise L, we need to maximize d^2

$$d^2 = \frac{4m^2+9}{m^2+1} = \frac{4(m^2+1)+5}{m^2+1} = 4 + \frac{5}{m^2+1}$$

$d^2_{max} = 4 + \frac{5}{0+1}$, since $m^2 \geq 0$

d_{max} occurs when $m=0$

$d^2_{max} = 9$

$L_{min} = 2\sqrt{25-9} = 2 \times 4 = 8$

D

24) <u>X-Intercept</u>

Any point on the x-axis can be represented by

$\bar{r} = x\hat{i}$

Substitute $\bar{r}$ into equation of plane

$(x\hat{i}) \cdot \bar{n} = d$

$x(\hat{i} \cdot \bar{n}) = d$, $x = \frac{d}{\hat{i} \cdot \bar{n}}$

Similarly $y = \frac{d}{\hat{j} \cdot \bar{n}}$, $z = \frac{d}{\hat{k} \cdot \vec{n}}$

C

25) For any real value x

$0 \leq \sin^{100} x \leq 1$

$0 \leq \cos^{100} x \leq 1$

$\sin^{100} x - \cos^{100} x \leq 1$

Equality holds iff

$\sin^{100} x = 1 \Rightarrow \sin x = \pm 1$

$\cos^{100} x = 0 \Rightarrow \cos x = 0$

$x = n\pi \pm \frac{\pi}{2}$ $(n \in I)$

C

26) $\bar{a} = \hat{i} + \hat{j} + \hat{k}$

$\bar{b} = \hat{i} - \hat{j} + \hat{k}$

$\bar{c} = \hat{i} + 2\hat{j} - \hat{k}$

$$\Delta = \begin{vmatrix} \bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b} & \bar{a}\cdot\bar{c} \\ \bar{b}\cdot\bar{a} & \bar{b}\cdot\bar{b} & \bar{b}\cdot\bar{c} \\ \bar{c}\cdot\bar{a} & \bar{c}\cdot\bar{b} & \bar{c}\cdot\bar{c} \end{vmatrix}$$

Using Property

31) $\Delta = [\bar{a}\ \bar{b}\ \bar{c}]^2$

STP = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix} = 4$

$\Delta = 4^2 = 16$ (D)

27) $f(x) = \left[\frac{x}{15}\right]\left[-\frac{15}{x}\right]$

for $x \in (0, 15)$

$\left[\frac{x}{15}\right] = 0$, $f(x) = 0$

for $x > 15$ $\left[-\frac{15}{x}\right] = -1$

$f(x) = -\left[\frac{x}{15}\right]$

$x \in (15, 30) \to f(x) = -1$

$x \in (30, 45) \to f(x) = -2$

$x \in (45, 60) \to f(x) = -3$

$x \in (60, 75) \to f(x) = -4$

$x \in (75, 90) \to f(x) = -5$

Range = $\{-5, -4, -3, -2, -1, 0\}$

$= 6$ (C)

28) Total Ways = ${}^{21}C_{10} + {}^{22}C_{10} + \dots {}^{30}C_{10}$

Using Hockey-Stick Identity

$$\sum_{r=k}^{n} {}^{r}C_{k} = {}^{n+1}C_{k+1}$$

Total Ways = $\sum_{r=10}^{30} {}^{r}C_{10} - \sum_{r=10}^{20} {}^{r}C_{10}$

$= {}^{31}C_{11} - {}^{21}C_{11}$

${}^{n}C_{r} = {}^{n}C_{n-r}$

$\Rightarrow {}^{31}C_{20} - {}^{21}C_{10}$ (B)

29) domain and range of both $f(x)$ & $g(x)$ are $[0, \infty)$

As output of $f(x) \in [0, \infty)$ strictly means $f(\text{input}) \geq 0$

$h(x) = f(g(x))$

$h(x) \geq 0 \ \forall x \geq 0$

$g(x)$ is decreasing
$f(x)$ is increasing $\Rightarrow$ $\forall x \geq 0$ $h(x) \leq h(0)$

given $h(0) = 0$, $h(x) \leq 0$

So $h(x) \geq 0$ from range of f

$h(x) \leq 0$ as h is decreasing

so $h(x) = 0 \ \forall x \in [0, \infty)$

so $p(x) = 0$ (B)

30) $$\frac{x^2}{f(k^2+2k+5)} + \frac{y^2}{f(K+11)} = 1$$

For ellipse to lie on x-axis

$$f(K^2+2K+5) > f(K+1)$$

As $f(x)$ is decreasing

$$K^2+2K+5 < K+11$$

$$K^2+K-6 < 0$$

$$(K+3)(K-2) < 0$$

$$K \in (-3, 2) \quad B$$

Q31) $2x^2y\frac{dy}{dx} = \tan(x^2y^2) - 2xy^2$

$2x^2y\frac{dy}{dx} + 2xy^2 = \tan(x^2y^2)$

$\hookrightarrow \frac{d}{dx}\left[x^2y^2\right] = \tan(x^2y^2)$

$\int \cot(x^2y^2)\, d(x^2y^2) = \int dx$

$\log|\sin(x^2y^2)|\, d(x^2y^2) = \int (dx)$

$\log[\sin(x^2y^2)] = x + C$

$x = 1,\ y = \frac{\sqrt{\pi}}{2} \qquad C = -1$

Hence Equation for the curve is

$\log(\sin(x^2y^2)) = x - 1$

$\sin(x^2y^2) = e^{x-1}$

Q32) $\int \frac{\left(\sqrt[3]{x + \sqrt{2 - x^2}}\right)\left(\sqrt[6]{1 - x\sqrt{2 - x^2}}\right)}{\sqrt[3]{1 - x^2}}$

$\rightarrow \sqrt{2 - x^2} = \sqrt{2 - 2\sin^2\theta} \quad [x = \sqrt{2}\sin\theta]$

$= \sqrt{2}\cos\theta.$

$\cancel{\rightarrow \sqrt[3]{x + \sqrt{2 - x^2}} = \sin\theta + \sqrt{2 - 2\sin^2\theta}}$

First term: $\sqrt{2}\sin\theta + \sqrt{2}\cos\theta = \sqrt{2}(\sin\theta + \cos\theta)$

So $(x + \sqrt{2 - x^2})^{1/3} = 2^{1/6}(\sin\theta + \cos\theta)^{1/3}$

Second term: $1 - x\sqrt{2 - x^2}$

$= 1 - (\sqrt{2}\sin\theta)(\sqrt{2}\cos\theta) =$

$= 1 - 2\sin\theta\cos\theta = 1 - \sin(2\theta)$

$\rightarrow 1 - \sin(2\theta) = (\cos\theta - \sin\theta)^2$

$\rightarrow (1 - x\sqrt{2 - x^2})^{1/6} = ((\cos\theta - \sin\theta)^2)^{1/6}.$

$\rightarrow (\cos\theta - \sin\theta)^{1/3}$

Numerator: $2^{1/6}(\sin\theta + \cos\theta)^{1/3}.$
$(\cos\theta - \sin\theta)^{1/3}$

~~Denominator~~

$= 2^{1/6}(\cos^2\theta - \sin^2\theta)^{1/3} = 2^{1/6}(\cos 2\theta)^{1/3}$

Denominator:

$\sqrt[3]{1 - x^2} = (1 - 2\sin^2\theta)^{1/3} = (\cos 2\theta)^{1/3}$

Integrand: $\frac{2^{1/6}\cos(2\theta)^{1/3}}{[\cos(2\theta)]^{1/3}} = 2^{1/6}$

$I = \int 2^{1/6}\, dx = 2^{1/6}x + C.$

33. $y^3 - 3y + x = 0$

$\Rightarrow x = 3y - y^3$

diff' $dx = 3dy - 3y^2 dy$

$dx = 3(1-y^2)dy$

$$\text{Area} = \int_a^b y\,dx = \int_a^b y\,3(1-y^2)\,dy$$

$$= -\int_a^b 3y(y^2-1)\,dy \rightarrow \text{method 1}$$

easier method to get to options → By parts

$$\text{Area} = \int_a^b \underset{\text{I}}{y}\,\underset{\text{II}}{dx} = yx\Big]_a^b - \int_a^b dy\,x$$

$$= yx\Big]_a^b - \int_a^b \frac{x\,dx}{3(1-y^2)}$$

$$= f(x)\,x\Big]_a^b + \int_a^b \frac{x\,dx}{3(1-f(x)^2)}$$

$$= \int_a^b \frac{x\,dx}{3(1-f(x))^2} + bf(b) - af(a)$$

Ans: B

34.

$\because x^3 + ax^2 + bx + c$ is divisible by $(1+x^2)$

the eq^n can be written as

$x^3 + ax^2 + bx + c = (1+x^2)(x+a)$

$\rightarrow x^3 + ax^2 + bx + c = x + a + x^3 + ax^2$

comparing the eq^n

$c = a \qquad b = 1$ (value fixed)

$a, b, c \in \{1, 2, 3, 4, \ldots, 10\}$

Total no. of polynomials possible

$$\left.\begin{array}{ll} b=1, & a=c=1 \\ b=1, & a=c=2 \\ \vdots & \\ b=1, & a=c=10 \end{array}\right\} \text{10 possible polynomials}$$

$\therefore$ Ans: 10 C

35.

$$f(x) = \left[\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}} \right]$$

$\therefore a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$

$$= \frac{(x^{1/3}+1)(x^{2/3}+1-x^{1/3})}{x^{2/3} - x^{1/3} + 1} = x^{1/3} + 1$$

Now : $\frac{x-1}{x - x^{1/2}} = \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)}$.

$$= \frac{\sqrt{x}+1}{\sqrt{x}} = 1 + \frac{1}{\sqrt{x}} = 1 + x^{-1/2}$$

$f(x) = (x^{1/3}+1) - (1 + x^{1/2}) = x^{1/3} - x^{-1/2}$

$$T_{r+1} = C_r^{15} (x^{1/3})^{15-r} (x^{-1/2})^r$$

$$= (-1)^r ({}^{15}C_r)\, x^{\frac{15-r}{3} - \frac{r}{2}}$$

$\frac{15-r}{3} - \frac{r}{2} = 0$, $30 - 2r - 3r = 0$

$5r = 30$

$r = 6$

$T_7 = (-1)^6 C_6^{15} = {}^{15}C_6 = 5005$

36. $f(x) = \frac{\tan(\pi[x-\pi])}{1+[x]^2}$.

① $[x-\pi]$:. This is always an integer so $\tan(Z\pi)$ which is always zero.

② Denominator : $1+[x]^2$ since $[x]^2$ is an integer so its square will also be an integer then we can say

$$1 + [x]^2 \geq 1$$

Numerator → Always Zero

Denominator → never zero

Hence $f(x) = \frac{0}{1+[x]^2} = 0$

→ constant function : $f(x) = 0$

$f'(x) = 0$ for any value of x.

option (A)

37. $p(x) = \prod_{r=1}^{2013} (x^2+r^2)$

$\ln(p(x)) = \sum_{r=1}^{2013} \ln(x^2+r^2)$

$\frac{p'(x)}{p(x)} = \sum_{r=1}^{2013} \frac{2x}{x^2+r^2}$

$\frac{1}{2} p'(x) = \left[\sum_{r=1}^{2013} \frac{x}{x^2+r^2}\right]\left[\prod_{r=1}^{2013} (x^2+r^2)\right]$

$\to \int_0^1 \frac{1}{2} p'(x)\,dx = \frac{1}{2}[p(x)]_0^1$

$\to \int_0^1 \frac{1}{2} p'(x)\,dx = \frac{1}{2}[p(1)-p(0)]$

$\to p(1) = \prod_{r=1}^{2013} (1+r^2)$

$\to p(0) = \prod_{r=1}^{2013} (0^2+r^2) = \prod_{r=1}^{2013} (r^2)$

$\hookrightarrow (2013!)^2.$

$\frac{1}{2}\left[\prod_{r=1}^{2013} (1+r^2) - (2013!)^2\right] = \frac{1}{2}\left[\prod_{r=1}^{2013} (1+r^2) - k^2\right]$

$\to k^2 = (2013!)^2$

$\to k = 2013!$

38. $\int_0^x (t^2 - 8t + 13)\,dt = \left[\frac{t^3}{3} - \frac{8t^2}{2} + 13t\right]_0^x.$

$\Rightarrow \frac{x^3}{3} - 4x^2 + 13x = x\sin\left(\frac{a}{x}\right)$

$\Rightarrow \frac{x^2}{3} - 4x + 13 = \sin\left(\frac{a}{x}\right)$

$\Rightarrow f(x) = \frac{x^3}{3} - 4x + 13.$

$f'(x) = \frac{2x}{3} - 4$ → condition for maxima.

$f'(x) = 0 \quad \frac{2x}{3} = 4 \Rightarrow \boxed{x = 6}.$

$f(6) = \frac{6^2}{3} - (4)(6) + 13 = 12 - 24 + 13 = \underline{\underline{1}}.$

$-1 \le \sin\left(\frac{a}{x}\right) \le 1$

$\sin\left(\frac{a}{6}\right) = 1$

$\sin(\theta) = 2n\pi + \frac{\pi}{2}$ $[n = 0, 1, 2 \ldots]$.

$\frac{a}{6} = 2n\pi + \frac{\pi}{2}$

$a = 6\left(2n\pi + \frac{\pi}{2}\right) = 12n\pi + 3\pi)$

$a = 3\pi$ (A)

39. $x^2+y^2-10x=0$

$(x-5)^2+y^2=5^2$

$C:(5,0)$ $R:5$

$\therefore \frac{x-x_1}{a}=\frac{y-y_1}{b}=-2\frac{(ax_1+by_1+c)}{a^2+b^2}$

$(x_1,y_1):(5,0)$, $[a=1, b=-1, c=3]$

$\frac{x'-5}{1}=\frac{y'-0}{1}=\frac{-2[1(5)-1(0)+3]}{1^2+(-1)^2}=-2(8)$

$=-8$

$\boxed{\begin{matrix}x'=-3\\ y=8\end{matrix}}$ → reflected centre
radius will still be 5.

$(x+3)^2+(y-8)^2=5^2$

→ $x^2+y^2+6x-16y+48=0$

$g=6$
$f=-16$ $\therefore g+f+c=6-16+48$
$c=48$ $=38.$

40. Base of the log must be positive first of all and not Equal to 0.

$|x+1|>0$ $x+1\neq 1$

$x\neq -1$ ↓ $x\neq -2, x\neq 0$

argument of log must be positive.

$3+2x-x^2>0$

$(x-3)(x+1)<0$

$\boxed{-1<x<3}$.

So $3+2x-x^2=(x-3)|x|$

$-1<x<3$, $x\neq 0$

C:1: $0<x<3$

$|x|=x$

→ $3+2x-x^2=(x-3)x$

→ $3+2x-x^2=x^2-3x,$

→ $2x^2-5x-3=0$, $(2x+1)(x-3)=0$

$x=-1/2, 3$

× ×

C:2 $-1<x<0$

$|x|=-x$

→ $3+2x-x^2=(x-3)(-x)$

→ $3+2x-x^2=-x^2+3x$

→ $3+2x=3x$

→ $x=3$ ×

→

Hence no values of x satisfy.

Q1. $y = f(e^x) + f(\ln|x|)$ $\quad f(x)$ is $(0,1)$

→ f is $0 <$ input < 1

$0 < e^x < 1 \rightarrow e^x < 1$

$\ln x < 0$

$\boxed{D_1 = (-\infty, 0)}$.

→ $0 < \ln|x| < 1$

$e^0 < |x| < e \rightarrow 1 < |x| < e$

$-e < x < -1$

$1 < x < e$

$\boxed{(-e,-1) \cup (1,e)}$.

intersection of Domains: ∴

$D = D_1 \cap D_2 = (-\infty, 0) \cap [(-e,-1) \cup (1,e)]$

Since $x < 0$

$(1,e)$ is Excluded.

Hence $(-e,-1)$

Q2.

x, y, z are digits from $\{0,1,2,\dots,9\}$

$x \neq 0$ ∴ $x \in \{1,2,3,\dots 9\}$

∵ $x < y$ & $z < y$

∴ y must be at least 2

∵ $x \geq 1$

∴ for a fixed value of y,

x has $(y-1)$ no. of choices ∵ $1 \leq x < y$

and z has y no. of choices ∵ $0 \leq z < y$

∴ Total no. of such integers is given by

$(y-1) \times y$ where $y = \{2, \dots 9\}$

$\therefore \sum_{y=2}^{9} (y-1)y = \sum_{y=2}^{9} (y^2 - y)$

$= \frac{y(y+1)(2y+1)}{6} - \frac{y(y+1)}{2}$

$-1 \qquad +1$

$= \frac{\cancel{9}^{3}\,(\cancel{10}^{5})(19)}{\cancel{6}_{2}} - \frac{9(\cancel{10}^{5})}{\cancel{2}}$

$= 285 - 45$

$= 240$ Ans B

43. Set A contain $1 \to 700$

$A = \{3, 6, 9 - - - - 699\}$

$n(A) = 233$

Odd → $[3, 9, 15 - - 699]$ → 117

Even → $[6, 12, 18 - - - 696]$ → 116

Set B contains $1 \to 300$ ∴ divisible 7.

$B = \{7, 14, 21 - - - 294\}$

$\frac{294}{7} = 42 = (n(B))$

Odd: $\{7, 21, 35 - - 287\}$, $n(B)_{odd} = 21$

Even: $\{14, 28, 42 - - 294\}$, $n(B_{even}) = 21$

(a, b) for ordered pair in C states that a+b must be Even number.

The sum of two Z is Even if Both are odd / Both are even.

C:1 $117 \times 21 = 2457$

C:2 $116 \times 21 = 2436$

Total $= 4893$

But subtract $a \neq b$ cases [∴ common terms]

$[21, 42, 63 - - - 294]$

$= \frac{294}{21} = 14$

$\to 4893 - 14 = 4879$ (A)

44. $ABA^{-1} = B^2$

$(ABA^{-1})^2 = (B^2)^2$

$(ABA^{-1})(ABA^{-1}) = B^4$

$AB(A^{-1}A)BA^{-1} = B^4$

$AB(I)BA^{-1} = B^4$

$AB^2A^{-1} = B^4$, $B^2 = ABA^{-1}$

$\to A(ABA^{-1})A^{-1} = B^4$

$\to A^2BA^{-2} = B^4$

$\to A^k B A^{-k} = B^{2k}$ $[A^5 = I]$ $k = 5$.

$\to A^5 B A^{-5} = B^{(2)^5}$

$\to IBI = B^{32}$

$\to B = B^{32}$

$\to BB^{-1} = B^{32}B^{-1}$

$\to \boxed{I = B^{31}}$.

$\boxed{28 < 31 < 32}$ ∴ Ans.

(B)

46.

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F

us. find: $\sin^{-1}(a\sqrt{1-b^2} + b\sqrt{1-a^2})$

$x = \sin^{-1} a + \sin^{-1} b$

LHS : is aP.

$S = \frac{1}{3} + \frac{x}{4} + \frac{x^2}{16} + \frac{x^3}{64} - - - -$

$S = \frac{1}{3} + \frac{x/4}{1-x/4} = \frac{1}{3} + \frac{x}{4-x}$

$S = \frac{4+2x}{3(4-x)} = \frac{2(8-3\pi)}{3(16+3\pi)}$.

$\rightarrow \frac{2+x}{4-x} = \frac{8-3\pi}{16+3\pi}$.

$\rightarrow 32 + 6\pi + 16x + 3\pi x = 32 - 12\pi - 8x + 3\pi x$

$\rightarrow 6\pi + 16x = -12\pi - 8x$

$\rightarrow x = -\frac{18\pi}{24} = -\frac{3\pi}{4}$

MAIN STEP:

$\sin^{-1}(a\sqrt{1-b^2} + b\sqrt{1-a^2})$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$A = \sin^{-1} a \quad B = \cos^{-1} b$

$\sin^{-1}(\sin(A+B))$ (which is $\sin^{-1}(\sin x)$).

$x = -\frac{3\pi}{4}$

$\rightarrow \sin(-\frac{3\pi}{4}) = -\frac{1}{\sqrt{2}}$

$\rightarrow \sin^{-1}(-\frac{1}{\sqrt{2}})$ is in within Range $[-\pi/2, \pi/2]$

$\sin^{-1}(-\frac{1}{\sqrt{2}}) = -\pi/4$

46. We know

$(l-m)^2 + (p-q)^2 = 9 \rightarrow (i)$

$(m-n)^2 + (q-r)^2 = 16 \rightarrow (ii)$

$(n-l)^2 + (r-p)^2 = 25 \rightarrow (iii)$

Let points $P_1(l, p)$

$P_2(m, q)$ and $P_3(n, r)$

From i, ii, iii $\rightarrow$

As we $3^2 + 4^2 = 5^2$

Hence $\Delta P_1 P_2 P_3$ represents a right angled triangle.

$\frac{1}{2} \Delta A =$ area of $\Delta P_1 P_2 P_3$

Area of $\Delta P_1 P_2 P_3 = \frac{1}{2} \times 3 \times 4 = 6$

Hence $\det A = 2 \times 6 = 12$

$(\det A)^2 = 144$

Hence Answer is option B)

47 Given

1. $f : (0,1) \rightarrow (0,1)$ is a diffrentiable function

2. $f'(x) \neq 0 \quad \forall x \in (0,1)$

3. $f(\frac{1}{2}) = \frac{\sqrt{3}}{2}$

4.
$$\lim_{t \to x} \frac{\int_0^t \sqrt{1-(f(s))^2}\,ds - \int_0^x \sqrt{1-(f(s))^2}\,ds}{f(t) - f(x)} = f(x)$$

Limit is in $\frac{0}{0}$ form.

Hence we apply L'Hôpital's Rule by diffrentiating the numerator and the denominator with respect to t

$$\frac{d}{dt}\left(\int_0^t \sqrt{1-(f(s))^2}\,ds - \int_0^x \sqrt{1-(f(s))^2}\,ds \right)$$

$= \sqrt{1-(f(t))^2}$ (By Leibnitz Integral rule)

$$\frac{d}{dt}(f(t) - f(x)) = f'(t)$$

$$\lim_{t \to x} \frac{\sqrt{1-(f(t))^2}}{f'(t)} = f(x)$$

Evaluating at $t = x$

$$\frac{\sqrt{1-(f(x))^2}}{f'(x)} = f(x)$$

$$\sqrt{1-(f(x))^2} = f(x)\,f'(x)$$

Let $u = f(x)$
$\Rightarrow du = f'(x)\,dx$

$$\int \frac{u}{\sqrt{1-u^2}}\,du = \int dx$$

Let $w = 1-u^2$
$dw = -2u\,du$

$$\int \frac{-1}{2\sqrt{w}}\,dw = x + c'$$

$$-\sqrt{1-u^2} = x + c$$

where c and c' are arbitary constants

$$1 - f(x)^2 = (x+c)^2$$

$$f(x) = \sqrt{1-(x+c)^2}$$

We know $f(1/2) = \frac{\sqrt{3}}{2}$

$$\frac{\sqrt{3}}{2} = \sqrt{1-\left(\tfrac{1}{2}+c\right)^2}$$

On solving we get 2 cases

Case 1 $\Rightarrow \frac{1}{2} + c = \frac{1}{2} \Rightarrow c = 0$

$$f(x) = \sqrt{1-x^2}$$

$$f\left(\tfrac{1}{4}\right) = \sqrt{1-\left(\tfrac{1}{4}\right)^2}$$

$$f\left(\tfrac{1}{4}\right) = \sqrt{\tfrac{15}{16}} = \frac{\sqrt{15}}{4}$$

Case 2

$\frac{1}{2} + c = -\frac{1}{2} \Rightarrow c = -1$

$$f(x) = \sqrt{1-(x-1)^2}$$

$$f\left(\tfrac{1}{4}\right) = \sqrt{\tfrac{7}{16}} = \frac{\sqrt{7}}{4}$$

The possible values are $\left\{\frac{\sqrt{7}}{4}, \frac{\sqrt{15}}{4}\right\}$.

48

For graph of $y = x^2 + 2(a+4)x - 5a + 64$ to lie entirely above x axis it must have no real roots. Hence Discriminant should be less then zero.

$$b^2 - 4ac < 0$$

$$(2(a+4))^2 - 4(-5a+64) < 0$$

$$4(a^2+8a+16) + 20a - 256 < 0$$

$$4a^2 + 52a - 192 < 0$$

$$a^2 + 13a - 48 < 0$$

$$(a+16)(a-3) < 0$$

$$-16 < a < 3$$

Given range of a

$a \in [-5, 30] = 36$ values

Favourable integers = 8

$\Rightarrow \{-5, -4, -3, -2, -1, 0, 1, 2\}$

Hence probability

$$p(E) = \frac{8}{36} = \frac{2}{9}$$

49 Let AC diagonal lie along X axis.

$A = (0,0,0)$, $C = (2a, 0, 0)$

as $|\vec{AC}| = 2a$

Similiarly in the orignal square B and D would be at $(a, a, 0)$ and $(a, -a, 0)$

After folding B and D move to $B = (a, a, 0)$

$D = (a, 0, a)$

Shortest Distance Calculation

$$d = \frac{|(\vec{C} - \vec{A}) \cdot (\vec{AB} \times \vec{CD})|}{|\vec{AB} \times \vec{CD}|}$$

$\vec{AB} = B - A = (a, a, 0)$

$\vec{CD} = D - C = (-a, 0, a)$

$\vec{AB} \times \vec{CD} = (a^2, -a^2, a^2)$

$|\vec{AB} \times \vec{CD}| = a^2\sqrt{3}$

$\vec{C} - \vec{A} = (2a, 0, 0)$

$(\vec{C} - \vec{A}) \cdot (\vec{AB} \times \vec{CD}) = 2a^3 + 0 + 0$

$$d = \frac{2a^3}{a^2\sqrt{3}} = \frac{2a}{\sqrt{3}}$$

50

$$I = \int \frac{1 - x^2}{\sqrt{x}\sqrt{(1+x^2)^3}}\,dx$$

Dividing by x^2

$$I = \int \frac{\frac{1-x^2}{x^2}}{\frac{\sqrt{x}}{x^2}(1+x^2)^{3/2}}\,dx$$

$$\Rightarrow I = \int \frac{\left(\frac{1}{x^2} - 1\right)}{\sqrt{x}\left(x + \frac{1}{x}\right)^{3/2}}\,dx$$

Putting $t = \frac{\sqrt{x}}{\sqrt{1+x^2}}$

$$\Rightarrow t^2 = \frac{x}{1+x^2}$$

$$2t\,dt = \frac{(1-x^2)}{(1+x^2)^2}\,dx$$

$$I = \int \frac{1-x^2}{(1+x^2)^2} \cdot \frac{(1+x^2)^{1/2}}{\sqrt{x}}\,dx$$

Note $\Rightarrow \frac{1}{t} = x + \frac{1}{x}$

$$I = \int 2\,dt = 2t + c$$

$I = 2\frac{x^{1/2}}{(1+x^2)^{1/2}} + C \therefore \alpha : \beta : \gamma = 4:1:1$

51) $\bar{a} = (x, y, z)$

$\bar{d} = (1, -1, 2)$

A) $\bar{a} = 2\hat{i} - 2\hat{j} - 2\hat{k}$

B) $\bar{a} = -2\hat{i} - 2\hat{j} + 2\hat{k}$

C) $\bar{a} = -2\hat{i} + 2\hat{j} - 2\hat{k}$

D) $\bar{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$

M1) As given $\bar{a}$ & $\bar{d}$ is perpendicular

$\bar{a} \cdot \bar{d} = 0$

By options we can check options

M2

$|a| = 2\sqrt{3}$

$x^2 + y^2 + z^2 = 12$ --- (1)

$\bar{a} \cdot \bar{d} = 0$

$x - y + 2z = 0$ ---- (2)

Angle b/w $\bar{a}$ & $\hat{j}$ is obtuse

$\bar{a} \cdot \hat{j} < 0 \rightarrow y < 0$

$|b| = |c|$, $|a| = 1$

$$\frac{\bar{a} \cdot \bar{b}}{|a||b|} = \frac{\bar{a} \cdot \bar{c}}{|a||c|}$$

$\Rightarrow z^2 + xy = 0$

$x^2 + y^2 - xy = 12$

$x^2 + y^2 - 2xy = 12 - xy$

$(x-y)^2 = 12 + z^2$

$4z^2 = 12 + z^2$ $\because x - y = -2z$ from (2)

$3z^2 = 12$, $z = \pm 2$

$xy = -z^2$, $xy = -4$

$x^2 + y^2 = 8$

$|x| = 2$, $|y| = 2$

As $y < 0$, $y = -2$

As $y = -2$, & $xy = -4$, hence $x = 2$

$x - y + 2z = 0$

$2 - (-2) = -2z$

$z = -2$

$\bar{a} = 2\hat{i} - 2\hat{j} - 2\hat{k}$ (A)

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52

6 sets $A_1, A_2 \dots A_6$ each with 4 elements.

n sets $B_1, B_2 \dots B_n$ each with 2 elements.

Each Element in S belongs to exactly 4 A sets and 3 B sets

Total elements from

$A = 6 \times 4 = 24$

(counting repeats)

If each distinct element appears exactly 4 times, the total number of elements N must satisfy

$4 \times N = 24 \Rightarrow N = 6$

Similiarly total elements from $B = 2n$

Hence

$3 \times N = 2n \Rightarrow 3 \times 6 = 2n$

We get $n = 9$

53 Given:

- Region bounded by $y = x^2+1$, $y=0$, $x=0$, $x=1$
- A tangent at point (x_0, y_0) on $y = x^2+1$ cuts the region into a trapezium

The tangent to $y = x^2+1$ at (x_0, x_0^2+1) has the equation

$y - (x_0^2+1) = 2x_0(x - x_0)$

$\Rightarrow y = 2x_0 x$

This line acts as the slanted side of the trapezium

The trapezium is formed by the vertices $(0,0), (1,0), (1, y(1)), (0, y(0))$

Height $h = 1$ distance between $x=0$ and $x=1$

$L_1 = -x_0^2 + 1 = y(0)$

$L_2 = 2x_0 - x_0^2 + 1 = y(1)$

$A(x_0) = \frac{L_1 + L_2}{2} \times h$

$\Rightarrow \frac{(-x_0^2+1) + (2x_0 - x_0^2+1)}{2}$

$\Rightarrow -x_0^2 + x_0 + 1 = A(x_0)$

To maximize $A(x_0)$

We take the derivative and set to zero

$$A'(x_0) = -2x_0 + 1 = 0$$

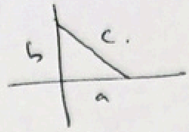
$$x_0 = \frac{1}{2}$$

$$y = x^2 + 1$$

$$y_0 = \frac{1}{4} + 1 = \frac{5}{4}$$

The point is $\left((1/2),(5/4)\right)$

(54) $a^2 + b^2 = c^2$



$a = c\cos\theta$.

$b = c\sin\theta$

$A(c\cos\theta, 0)$.

$B(0, c\sin\theta)$

$P(c\cos\theta, c\sin\theta)$.

$$\frac{x}{c\cos\theta} + \frac{y}{c\sin\theta} = 1$$

$$x\sin\theta + y\cos\theta = c\sin\theta\cos\theta.$$

slope of AB : $-\tan\theta$

So the slope of the perpendicular PM is $\cot\theta = \frac{\cos\theta}{\sin\theta}$.

$$y - c\sin\theta = \frac{\cos\theta}{\sin\theta}(x - c\cos\theta).$$

$$y\sin\theta - c\sin^2\theta = x\cos\theta - c\cos^2\theta.$$

$$x\cos\theta - y\sin\theta = c(\cos^2\theta - \sin^2\theta) \to \text{eq}(2)$$

$$(x\sin^2\theta + y\sin\theta\cos\theta) + (x\cos^2\theta - y\sin\theta\cos\theta)$$
$$= c\sin^2\theta\cos\theta + c\cos\theta(\cos^2\theta - \sin^2\theta)$$

$$\to x(\sin^2 + \cos^2\theta) = c\sin^2\theta\cos\theta + c\cos^3\theta - c\sin^2\theta\cos\theta$$

$$x = c\cos^3\theta.$$

$$y = c\sin^3\theta.$$

$$(\cos\theta)^3 = \frac{x}{c} \qquad \cos\theta = \left(\frac{x}{c}\right)^{\frac{1}{3}}$$

$$(\sin\theta)^3 = \frac{y}{c} \qquad \sin\theta = \left(\frac{y}{c}\right)^{\frac{1}{3}}$$

$$\left[\left(\frac{x}{c}\right)^{\frac{1}{3}}\right]^2 + \left[\left(\frac{y}{c}\right)^{\frac{1}{3}}\right]^2 = 1$$

$$x^{2/3} + y^{2/3} = c^{2/3}$$

55

Roots of $y^2 - my + 1 = 0$ are r_1 and r_2. 1 lies between them.

We take $r_1 < 1 < r_2$

Now if 1 lies between the roots then $F(1) < 0$

$$F(1) = 1 - m + 1 < 0$$

$$2 < m$$

We want the value of $\left[\left(\frac{4|x|}{x^2+16} \right)^m \right]$

~~AM ≤ GM~~

$$x^2 + 16 \geq 2\sqrt{x^2 16} = 8|x|$$

So, putting AM ≥ GM

$$\frac{4|x|}{x^2+16} \leq \frac{4|x|}{8|x|} = \frac{1}{2}$$

$$\left(\frac{4|x|}{x^2+16} \right)^m \leq \left(\frac{1}{2} \right)^m$$

we know $m > 2$

$$\Rightarrow \left(\frac{1}{2} \right)^2 = \frac{1}{4}$$

Box of $\left[\frac{1}{4} \right] = 0$

Answer (C) 0

56

Given

→ $F(x)$ is twice diffrentiable in $[1, 3]$.

→ $F(1) = F(3)$

→ $|F''(x)| \leq 2$

By Mean Value Theorem on interval $[1,2]$:

There exists a point $c_1 \in (1, 2)$

$$F'(c_1) = \frac{F(2) - F(1)}{2-1}$$

Similiarly for $[2, 3]$

$c_2 \in (2, 3)$ such that

$$F'(c_2) = \frac{F(3) - F(2)}{3-2}$$

Since $(F(1)) = (F(3))$

$$F'(c_1) = -F'(c_2)$$

Hence slope at some point in $[1,2]$ is negative of slope at $[2,3]$

As $|F''(x)| \leq 2$

$F'(x)$ cannot change faster than 2 units

$$|F'(x_b) - F'(x_a)| \leq |x_b - x_a| \times |F''(x)|$$

9).

Thus for any $x \in [1,3]$

$$|f'(x) - f'(c)| \le 2|x-c|$$

By Rolle's Theorem

$$f'(c) = 0$$

$$|f'(x)| \le 2|x-c|$$

Between any x and c maximum distance is 2 in [1,3]

Hence

$$|f'(x)| \le 2 \times 2 = 4$$

(57) $f(r) = 6r^4 - 16r^3 - 3r^2 + 12r.$

$f'(r) = 24r^3 - 48r^2 - 6r + 12$

$\to 4r^3 - 8r^2 - r + 2 = 0.$

$\to (2r-1)(2r+1)(r-2).$

$r = \frac{1}{2}.$

(58) $f(x+a) = \frac{1}{2} + \sqrt{f(x) - f^2(x)}$

$f(x+2a) = \frac{1}{2} + \sqrt{f(x+a) - [f^2(x+a)]}$

$f(x+a) - f^2(x+a)$

$\to \left[\frac{1}{2} + \sqrt{f(x) - f^2(x)}\right] - \left[\frac{1}{2} + \sqrt{f(x) - f^2(x)}\right]^2$

$\sqrt{f(x) - f^2(x)} = A.$

$\left(\frac{1}{2} + A\right) - \left(\frac{1}{2} + A\right)^2.$

$= \frac{1}{2} + A - \left(\frac{1}{4} + A + A^2\right)$

$= \frac{1}{4} - A^2$

$= \frac{1}{4} - [f(x) - f^2(x)]$

$= \frac{1}{4} - f(x) + f^2(x)$

$= \left[f(x) - \frac{1}{2}\right]^2$

$f(x+2a) = \frac{1}{2} + \sqrt{\left(f(x) - \frac{1}{2}\right)^2}$

$f(x+2a) = \frac{1}{2} + \left|f(x) - \frac{1}{2}\right|$

$\left|f(x) - \frac{1}{2}\right| = f(x) - \frac{1}{2}$

$f(x+2a) = \frac{1}{2} + f(x) - \frac{1}{2}$

$f(x) = f(x+2a)$ (D) $\to 2a$: Ans.

(59) natural numbers 10 digits $(0 \to 9)$.

Total $\to$ 10 choices for single digits.

For products of four number to have a unit's digit 1, 3, 7, 9.

$\to$ No number can end in an even digit. (0, 2, 4, 6, 8)

$\to$ No number can end in 5.

Therefore in units digit of each of the four selected number must belong strictly to the set [1, 3, 7, 9)

$= \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{16}{125}.$

(60) $y = f^{-1}(x)$

$x = f(y)$

$dx = f'(y)\,dy.$

$x = f(a),\ y = f^{-1}(f(a)) = a.$

$x = f(b),\ y = f^{-1}(f(b)) = b$

$I = \int_{f(a)}^{f(b)} 2x\{b - f^{-1}(x)\}\,dx.$

$I = \int_a^b (b-y) \cdot 2f(y) f'(y)\,dy$

$I = \int_a^b (b-x)\, 2f(x) f'(x)\,dx.$

$\frac{d}{dx}(f^2(x)) = 2f(x) f'(x)$

$I = \int_a^b (b-x) \frac{d}{dx}(f^2(x))$

$dv = u \cdot v - \int v\, du$

let $u = b - x \rightarrow du = -dx$

$dv = \frac{d}{dx}(f^2(x))\, dx \Rightarrow v = f^2(x)$

$I = \left[(b-x) f^2(x)\right]_a^b - \int_a^b f^2(x)(-1)\, dx$

$I = \{(b-b) f^2(b) - (b-a) f^2(a)\} + \int_a^b f^2(x)\, dx$

$I = 0 - (b-a) f^2(a) + \int_a^b f^2(x)\, dx$

$-(b-a) f^2(a)$

$-(b-a) f^2(a) = -\int_a^b f^2(a)\, dx.$

$I = \int_a^b f^2(x)\, dx - \int_a^b f^2(a)\, dx$

$I = \int_a^b [f^2(x) - f^2(a)]\, dx$

$\hookrightarrow (x)$

(6)

$y = x^3$

$\frac{dy}{dx} = 3x^2$

$P_1(x_1, y_1)$ or $P_1(x_1, x_1^3)$

Slope at $P_1 = 3x_1^2$

$y - x_1^3 = 3x_1^2(x - x_1)$

$y = 3x_1^2 x - 2x_1^3$

P_2 $\boxed{y = x^3}$.

$\hookrightarrow (x_2, y_2)$

$x^3 = 3x_1^2 x - 2x_1^3$

$x^3 - 3x_1^2 x + 2x_1^3 = 0$

The line is tangent to the curve at $x = x_1 \rightarrow$ it must be a repeated root. let third root be x_2.

using rules of cubic,

$x_1 + x_2 + x_3 = 0$

$x_2 = -2x_1$

$x_3 = -2x_1 = -2(-2x_1) = 4x_1$

$x_n = -2x_{n-1}$

$\frac{x_2}{x_1} = \frac{x_3}{x_2} - - - - - = -2.$

Hence (G.P) with $r = -2$

62

$x^3 + 5x^2 + px + q = 0 \to (i)$

$x^3 + 7x^2 + px + r = 0 \to (ii)$

Assume they have common roots α, β.

For sum of roots in cubic equation

$x^3 + ax^2 + bx + c = 0$

sum of roots $= -a$

$\alpha + \beta + x_1 = -5$ from (i)

$\alpha + \beta + x_2 = -7$ from (ii)

$\Rightarrow x_2 = x_1 - x_2$

For multiplication of roots taking at 2 at a time in cubic equation

$x^3 + ax^2 + bx + c = 0$

$b = \alpha\beta + \beta\gamma + \gamma\delta$

where α, β, δ are roots

Thus

$p = \alpha x_1 + \beta x_1 + \alpha\beta$ from (i)

$p = \alpha x_2 + \beta x_2 + \alpha\beta$ from (ii)

~~Dividing~~ subtracting equations

$\alpha(x_1 - x_2) = \beta(x_2 - x_1)$

$\alpha = -\beta$.

Putting $\alpha = -\beta$ in

$\alpha + \beta + x_1 = -5$

$x_1 = -5$

similiarly

$x_2 = -7$

Then G.C.D of x_1, x_2 is clearly 1.

63 Given

$$\int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx$$

$$=\int_0^2 (1+\cos^8 x)(ax^2+bx+c)dx$$

Splitting the integral

$$\int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx$$

$$=\int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx + \int_1^2 (1+\cos^8 x)(ax^2+bx+c)dx$$

$$\int_1^2 (1+\cos^8 x)(ax^2+bx+c)dx = 0$$

As $1+\cos^8 x > 0$ always hence ax^2+bx+c most change sign in 1,2.

Hence For K in [1,2] $g(K)=0$ most be there.

Hence atleast one root in (1,2)

64 z_1, z_2 are roots of the quadratic equation

$$z^2+pz+q=0$$

$|z_1| = |z_2|$ as complex roots exist in conjugate pairs.

Taking

$$z_1 = re^{i\theta}$$

$$z_2 = re^{i(\theta+\alpha)}$$

Sum of roots $= -p$

$$-p = re^{i\theta}(1+^{i\alpha})^2$$

Product of roots $= q$

$$q = r^2 e^{i(2\theta+\alpha)}$$

$$p^2 = r^2 e^{i2\theta}(1+e^{i\alpha})^2$$

$$\underbrace{1+e^{i\alpha} = 2\cos\left(\frac{\alpha}{2}\right) e^{i\alpha/2}}_{\text{Identity}}$$

$$p^2 = 4r^2\cos^2\frac{\alpha}{2}\, e^{i(\theta+\alpha)}$$

$$\frac{p^2}{q} = 4\cos^2\frac{\alpha}{2}$$

$$\frac{p^2}{q}\sec^2\frac{\alpha}{2} = 4\times 1 = 4$$

(C) 4

Q65

$g(x) = ax + b$
with $a < 0$

~~$g(x)$~~

$\cot\left(\cos^{-1}(|\sin x| + |\cos x|) + \sin^{-1}(-|\cos x| - |\sin x|)\right)$

$|\sin x| + |\cos x| = \theta$

$\cot(\cos^{-1}(\theta) + \sin^{-1}(\theta))$

$\cot(\frac{\pi}{2}) = 0$

$g(3) = 3a + b = 0$
$3a = -b$

$g(1) = (a+b) = 2$

$a - 3a = 2$

$a = -1$

$b = -3(-1) = 3$

$g(x) = -x + 3$

$g(3) = 0$

We check at $g(3) = 0$ for our required answer

Q66

Given

$$\sum_{r=0}^{2n} a_r (x-2)^r = \sum_{r=0}^{2n} b_r (x-3)^r$$

$$x - 2 = y + 1$$

$$\sum_{r=0}^{2n} a_r (y+1)^r = \sum_{r=0}^{2n} b_r y^r$$

$a_r = 1$ for $r \geq 1$

$$f(x) = a_0 + \sum_{r=1}^{2n} (y+1)^r$$

For a term $(y+1)^r$ y^n is given by ${}^rC_n$.

b_n is defined as the coefficient of $(x-3)^n$ which is coefficient of y^n

Thus

$$b_n = \sum_{r=n}^{2n} {}^rC_n$$

By Hockey stick

$$b_n = {}^{2n+1}C_{n+1}$$

$$\frac{b_n}{{}^{2n+1}C_{n+1}} = 1$$

67)

$$x = \lim_{n\to\infty} \frac{[1^2 f(x)^x] + [2^2 f(x)^x] + [3^2 f(x)^x] \cdots]}{n^3}$$

$$a-1 < [a] \le a$$

$$\sum_{k=1}^{n} (k^2 f(x)^x - 1) < \sum_{k=1}^{n} [k^2 f(x)^x] \le \sum_{k=1}^{n} k^2 f(x)^x$$

$$f(x)^x \sum_{k=1}^{n} k^2 - n < \sum_{k=1}^{n} [k^2 f(x)^x] \le f(x)^x \sum_{x=1}^{n} k^2$$

$$\sum k^2 = \frac{k(k+1)(2k+1)}{6}$$

$$\frac{f(x)^x \frac{n(n+1)(2n+1)}{6} - n}{n^3} < \frac{\sum_{k=1}^{n} [k^2 f(x)^x]}{n^3}$$

$$\le \frac{f(x)^x \frac{n(n+1)(2n+1)}{6}}{n^3}$$

• LHL

$$\lim_{n\to\infty} \frac{f(x)^x (2n^3 + 3n^2 + n) - 6n}{6n^3} = \frac{2}{6} f(x)^x$$

$$= \frac{1}{3} f(x)^x$$

RHL

$$\lim_{n\to\infty} \frac{f(x)^x (2n^3 + 3n^2 + n)}{6n^3} = \frac{2}{6} f(x)^x = \frac{1}{3} f(x)^x$$

$$x = \frac{1}{3} f(x)^x$$

$$f(x) = (3x)^{1/x}$$

$$\log f(x) = \log((3x)^{1/x}).$$

$$\log f(x) = \frac{1}{x} \log(3x)$$

$$\frac{f'(x)}{f(x)} = \frac{x \frac{d}{dx}(\log 3x) - \log(3x) \frac{d(x)}{dx}}{x^2}$$

$$\frac{f'(x)}{f(x)} = \frac{x\left(\frac{1 \times 3}{3x}\right) - \log(3x)}{x^2}.$$

$$\frac{f'(x)}{f(x)} = \frac{1 - \log(3x)}{x^2}.$$

$$f'(x) = (3x)^{1/x} \left[\frac{1 - \log 3x}{x^2}\right]$$

$$f'(x) = \frac{1 - \log(3x)}{x^2}$$

$$f'(x) = f(x) \left[\frac{1 - \log 3x}{x^2}\right]$$

$$f'(x) = (3x)^{1/x} \left[\frac{1 - \log 3x}{x^2}\right].$$

(61.) $x-y\, f(x+y) - (x+y) f(x-y) = 2(x^2y - y^3)$

$2(x^2y - y^3) = 2y(x^2 - y^2) = 2y(x-y)(x+y)$

$(x-y) f(x+y) - (x+y) f(x-y) = 2y(x-y)(x+y)$

$x \neq \pm y$

$\rightarrow \frac{f(x+y)}{x+y} - \frac{f(x-y)}{x-y} = 2y$

$x+y = u \qquad u - v = 2y$

$x-y = v$

$\rightarrow \frac{f(u)}{u} - \frac{f(v)}{v} = u - v$

$\rightarrow \frac{f(u)}{u} - u = \frac{f(v)}{v} - v$

$\rightarrow \frac{f(x)}{x} = x + c \Rightarrow f(x) = x^2 + cx.$

$f(1) = 2.$

$\hookrightarrow f(1) = 1^2 + c(1) = 2$

$c = 1$

$\boxed{f(x) = x^2 + x}.$

A ✓ → polynomial

$f(3) = 12$

$f(0) = 0$

69.

$$\lim_{n\to\infty} \sum_{r=1}^{n} a^{r-1} \underbrace{\int_{(r-1)a}^{ra} \frac{f(x)\,dx}{f(x) + f(2ra - a - x)}}_{I_1} = \frac{3}{5}.$$

Kings rule

$$I_2 = \int \frac{f(2ra-a-x)}{f(2ra-a-x) + f(x)}.$$

$I_1 = I_2.$

$. \ I_1 + I_2 = 2I_1.$

$$2I_1 = \int_{(r-1)a}^{ra} \cancel{\frac{f(x) + f(2ra-a-x)}{f(x) + f(2ra-a-x)}}\, dx.$$

$2I_1 = a.$

$I_1 = \frac{a}{2}$

$\frac{1}{2}\left[\sum_{r=1}^{r=n} a^r\right] \rightarrow \frac{1}{2}\left(\frac{a}{1-a}\right) = \frac{3}{5}.$

$5a = 6 - 6a$

$\boxed{a = \frac{6}{11}}.$

(70) $x = 1-3t^2$

$y = t-3t^3$

$P(-2,2)$

$-2 = 1-3t^2, \; t = \pm 1$

if $t=1 \Rightarrow y = 1-3(1)^3 = -2$

$\underline{\underline{t=-1}} \Rightarrow y = (-1)-3(-1)^3 = 2.$

<u>checking symmetry:</u>

$\to x(-t) = 1-3(-t)^2 = 1-3t^2$

$\to y(-t) = (-t)-3(-t)^3 = -t+3t^3$
$= -(t-3t^3) = -y(t)$

(A) ✓
↳ symmetric about x-axis.

$\tan\phi = \dfrac{dy}{dx}$

$\dfrac{dx}{dt} = -6t, \quad \dfrac{dy}{dt} = 1-9t^2$

$\tan\phi = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{1-9t^2}{-6t} = \dfrac{9t^2-1}{6t}$

$= \sqrt{1+\tan^2\phi}$

$\sec\phi = \sqrt{1+\left(\dfrac{9t^2-1}{6t}\right)^2}$

$= \sqrt{\dfrac{36t^2+(81t^4-18t^2+1)}{36t^2}}$

$= \sqrt{\dfrac{(9t^2+1)^2}{(6t)^2}} = \dfrac{9t^2+1}{6t}$

$\tan\phi + \sec\phi = \dfrac{9t^2-1}{6t} + \dfrac{9t^2+1}{6t} = \dfrac{18t^2}{6t} = 3t$

(C) ✓

$\frac{1}{\sqrt{10}})$

$\sqrt{\frac{10}{11}}$

$\frac{0}{11}$

$(\frac{21}{11})^2 + 3300(\frac{21}{11})$

$+3300$

$0(\frac{10}{11})+3300$

$^2 + 3300(\frac{10}{11})$

$(11x-10)^3$

Slope at point P $(t_1 = -1)$

$$m_1 = \frac{9(-1)^2 - 1}{6(-1)} = \frac{8}{-6} = \frac{-4}{3}$$

$P(-2, 2)$

$$y - 2 = -\frac{4}{3}(x + 2) \rightarrow 4x + 3y + 2 = 0$$

$$\rightarrow 4(1 - 3t^2) + 3(t - 3t^3) + 2 = 0$$

$$\rightarrow 4 - 12t^2 + 3t - 9t^3 + 2 = 0.$$

$$\rightarrow 9t^3 + 12t^2 - 3t - 6 = 0$$

$$\rightarrow (t+1)^2 (9t - 6) = 0.$$

$9t - 6 = 0 \quad t_2 = \frac{2}{3}$

$t_2 = \frac{2}{3}$

$$m_2 = \frac{9\left(\frac{2}{3}\right)^2 - 1}{6\left(\frac{2}{3}\right)} = \frac{4-1}{4} = \frac{3}{4}.$$

$$m_1 m_2 = \left(-\frac{4}{3}\right)\left(\frac{3}{4}\right) = -1$$

71) $f(x) = x\left(1331x^2 - 3630x + 3300\right)$

$a = \cos^2\left(\tan^{-1}\left(\sin\left(\cot^{-1}(3)\right)\right)\right)$

$= \cos^2\left(\tan^{-1}\left(\sin\left(\sin^{-1}\frac{1}{\sqrt{10}}\right)\right)\right)$

$= \cos^2\left(\tan^{-1}\left(\frac{1}{\sqrt{10}}\right)\right)$

$= \left(\cos\left(\cos^{-1}\frac{\sqrt{10}}{\sqrt{11}}\right)\right)^2$

$a = \frac{10}{11}$

$f(x) = \underbrace{1331x^3 - 3630x^2 + 3300x - 1000}_{(11x-10)^3} + 1000$

$f(x) = (11x-10)^3 + 1000$

A) $f(a+1) = \left(\cancel{11} \times \frac{21}{\cancel{11}} - 10\right)^3 + 1000 = 1331 + 1000 = 2331$

B) $f'(a) = 3(11x-10)^2 \cdot 11 = 0$

C) $f(a) = \left(11 \times \frac{10}{11} - 10\right)^3 + 1000 = 1000$

D) $\int_0^a f(x) - 1000 = \int_0^{10/11} (11x-10)^3 dx = \frac{(11x-10)^4}{4 \times 11} = -\frac{2500}{11}$

Answer = A, C

(72)

Since $\bar{r}$ is $\perp^r$ to $\bar{a}+\bar{b}+\bar{c}$

$\bar{r}.(\bar{a}+\bar{b}+\bar{c})=0$

substitute $\bar{r}$ & expand

$\left(\sin x(\bar{a}\times\bar{b})+\cos y(\bar{b}\times\bar{c})+2(\bar{c}\times\bar{a})\right).(\bar{a}+\bar{b}+\bar{c})=0$

$\sin x\,[\bar{a}\;\bar{b}\;\bar{c}]+\cos y\,[\bar{b}\;\bar{c}\;\bar{a}]+2[\bar{c}\;\bar{a}\;\bar{b}]=0$

As $[\bar{a}\;\bar{b}\;\bar{c}]=[\bar{b}\;\bar{c}\;\bar{a}]=[\bar{c}\;\bar{a}\;\bar{b}]$

$\sin x+\cos y+2=0$

Only possible ~~for~~ when

$\sin x=-1,\ \cos y=-1$

$x=2n\pi-\frac{\pi}{2}$

$y=(2m+1)\pi$

let $n=0,\ m=0$

$x^2+y^2=\frac{5\pi^2}{4}$ (A)

Let $n=0,\ m=1$

$x^2+y^2=\frac{37\pi^2}{4}$ (C)

A, C

(73)

$y=4-x^2$

$V:(0,4)\qquad x=\pm 2.$

$A:(-2,0)\quad B:(2,0)$

The vertex $(0,4)$ moves on $y=x+4$

new coordinates satisfy:

$k=h+4.$

$y-k=-(x-h)^2.$

$y-(h+4)=-(x-h)^2$

(2,0) $0-(h+4)=-(2-h)^2$

$-h-4=-[4-4h+h^2]$

$-h-4=-4+4h-h^2$

$h^2-5h=0$

$h(h-5)=0$

new: $(5,9)$

$y-9=-(x-5)^2$

$0-9=-(x-5)^2$

$x-5=\pm 3$

$x=2$ (8)

(75)

$$\left((4)^{\sec^2\alpha}\right)x^2 + 2x + \left(\beta^2 - \beta + \tfrac{1}{2}\right) = 0$$

$$A = 4^{\sec^2\alpha} \;|\; B = 2 \;|\; C = \beta^2 - \beta + \tfrac{1}{2}$$

$$D \geq 0$$

$$B^2 - 4AC \geq 0$$

$$4\left(1 - \left(4^{\sec^2\alpha}\right)\left(\beta^2 - \beta + \tfrac{1}{2}\right)\right) \geq 0$$

$$\left(4^{\sec^2\alpha}\right)\left(\beta^2 - \beta + \tfrac{1}{2}\right) \leq 1$$

Now,

$$\sec^2\alpha \geq 1 \Rightarrow 4^{\sec^2\alpha} \geq 4$$

$$\left(\beta^2 - \beta + \tfrac{1}{2}\right) = \left(\beta - \tfrac{1}{2}\right)^2 + \tfrac{1}{4} \geq \tfrac{1}{4}$$

So, both factor must at their minimum

$$\Rightarrow \sec^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 \Rightarrow \cos\alpha = \pm 1$$

$$\cos\alpha = 1 \text{ at } \alpha = 2n\pi$$

$$\cos\alpha = -1 \text{ at } \alpha = (2n+1)\pi$$

and,

$$\beta - \tfrac{1}{2} = 0 \Rightarrow \beta = \tfrac{1}{2}$$

$$\cos^{-1}\left(\tfrac{1}{2}\right) = \pi/3.$$

$$\cos\alpha + \cos^{-1}(\beta) = 1 + \pi/3 \left(\text{for even } \pi\right)$$

(A)

$$= -1 + \pi/3 \left(\text{for odd } \pi\right)$$

(C)

Ans → (A) (C)

(74)

$$P(\text{none occur}) = \prod_{i=1}^{1006}\left(1 - P(A_i)\right)$$

$$= \prod_{i=1}^{1006}\left(1 - \tfrac{1}{2i}\right)$$

$$= \prod_{i=1}^{1006}\left(\frac{2i-1}{2i}\right)$$

$$= \frac{1.3.5.\ldots.2011}{2.4.6.\ldots.2012}$$

$$\Rightarrow \frac{(2012)!}{(2.4.6.\ldots 2012)^2}$$

$$= \frac{(2012)!}{\left(2^{1006}.1006!\right)^2}$$

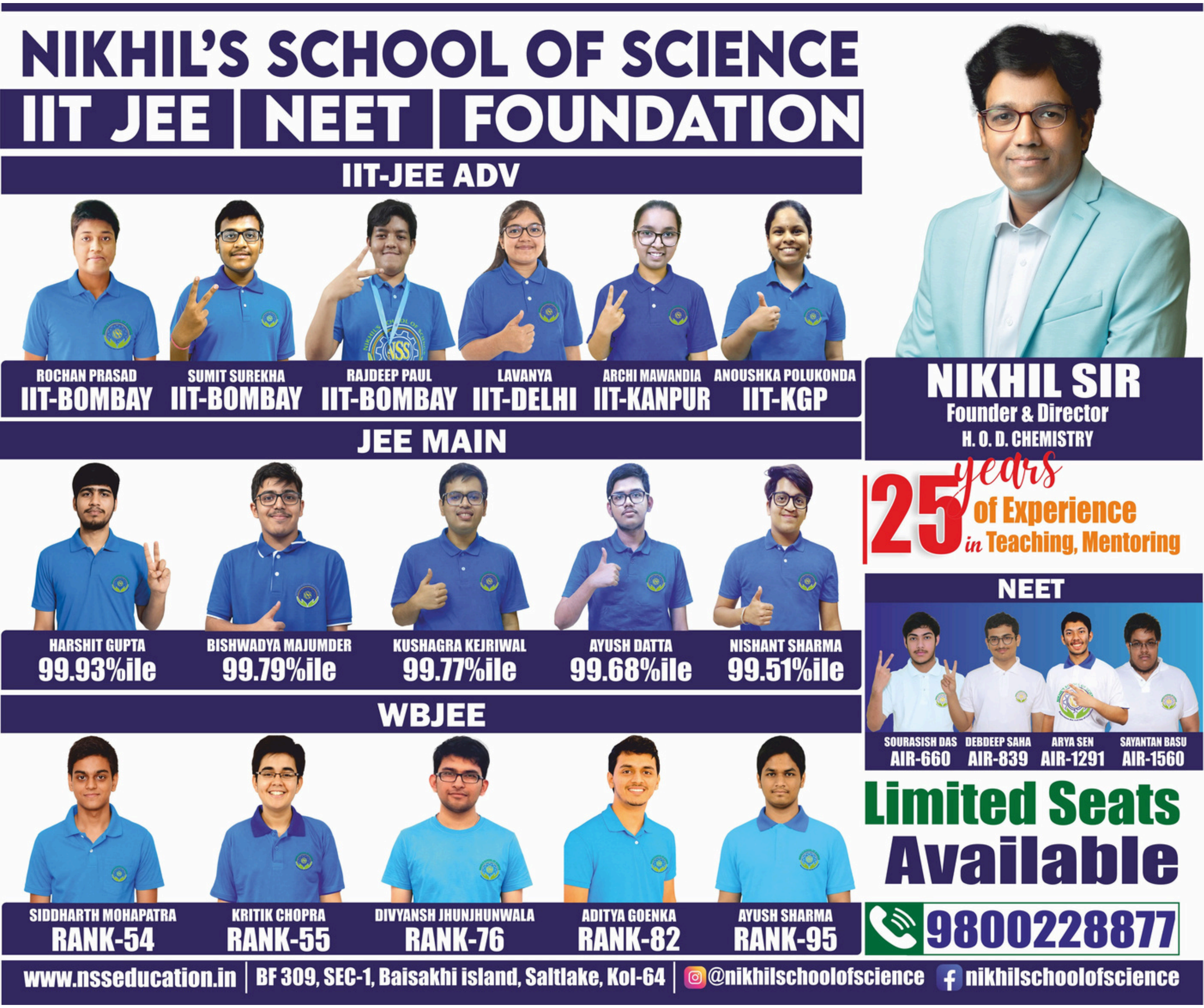
$$= \frac{\alpha!}{2^{\alpha}(\beta!)^2}$$

$$\alpha = 2012 \;|\; \beta = 1006$$

$$\beta = 1006 = 4(251) + 2 \quad \text{(A)}$$

$$\alpha = 2(1006) = 2\beta \quad \text{(B)}$$

Ans → (A) (B).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